Quantum kinetic theory of plasmas in strong laser fields

D. Kremp, Th. Bornath, and M. Bonitz

Fachbereich Physik, Universität Rostock, Universitätsplatz 3, D-18051 Rostock, Germany

M. Schlanges

Institut für Physik, Ernst-Moritz-Arndt Universität Greifswald, Domstrasse 10a, 17487 Greifswald, Germany

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A kinetic theory for quantum many-particle systems in time-dependent electromagnetic fields is developed based on a gauge-invariant formulation. The resulting kinetic equation generalizes previous results to quantum systems and includes many-body effects. It is, in particular, applicable to the interaction of strong laser fields with dense correlated plasmas. [S1063-651X(99)10609-3]

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I. INTRODUCTION

Recent impressive developments in the field of short-pulse laser technology [1] make it possible to create strongly correlated plasmas in extreme nonequilibrium situations [2]. At the same time, optical techniques for time-resolved diagnostics are becoming available [3] which creates the need for theoretical modeling of dense nonideal plasmas in intense laser fields.

The conventional approach to nonequilibrium properties of plasmas is based on kinetic equations of the Boltzmann type. In spite of their fundamental character, Boltzmann-like kinetic equations have a number of shortcomings, especially with respect to dense plasmas in laser fields. Boltzmann-like equations are valid only for times larger than the correlation time. Further, the ordinary Boltzmann equation conserves only the mean kinetic energy instead of the sum of kinetic and potential energy. Finally, the collision integrals are independent of the electromagnetic field. However, strong correlations, high-frequency electromagnetic fields and short-time phenomena require generalizations. Kinetic equations for classical plasmas in high-frequency fields have been derived in the papers of Silin [4], Oberman *et al.* [5], and others, e.g., Ref. [6], and have been applied to the computation of the high-frequency electrical conductivity [7,6]. Collision frequencies were calculated also in a dielectric [8] and a ballistic [9] model, respectively. More recently, kinetic properties of dense quantum plasmas in strong static fields have been investigated [10,11]. Silin and Uryupin [12] developed a quantum approach to absorption in strong electromagnetic fields. However, a kinetic equation for dense quantum plasmas in time-dependent fields which is valid on arbitrary time scales is still missing.

Generalizations of the Boltzmann kinetic equation should include the following features. (i) As a consequence of strong correlations, the collision integral will be non-Markovian, i.e., it should contain collisional broadening and memory (retardation) effects, see, e.g., Refs. [6,13–16]. (ii) The electric field gives rise to an additional collisional broadening. Furthermore it yields an additional retardation in the distribution functions known as intracollisional field effect. (iii) Non-Markovian effects should be of particular importance for high-frequency fields varying on time scales shorter than the correlation time (essentially one over the plasma frequency) [4,6]. (iv) In high-intensity laser fields, it is easy to generate situations in which the quiver velocity $v_0 = eE_0/m\Omega$ is large in comparison to the thermal velocity $v_{th} = \sqrt{kT/m}$. Therefore our goal is to derive a field dependent collision integral which is valid for arbitrary ratios v_0/v_{th}. As was shown first by Silin [4], such collision integrals depend on the field strength in nonlinear way. Consequently, Coulomb scattering processes will give rise to the excitation of higher harmonics. Furthermore, Coulomb collision processes in intense fields can involve absorption and emission of photons (inverse bremsstrahlung and bremsstrahlung, respectively). On the other hand, for low frequencies and weak fields ($v_0/v_{th}<1$) one enters the linear response regime. In contrast to the usual linear response theory, however, one gets from the collision integral an additional term linear in the field which is known as the relaxation field contribution [17–19,10].

The importance of quantum effects can be estimated considering the ratios of the thermal wave length $\lambda = h/\sqrt{2\pi mkT}$, which characterizes the extension of the probability density of the plasma particles, to other characteristic lengths. Quantum effects are to be expected (i) if the Landau length $l = e^2/kT$ is of the same order as the thermal wave length, i.e., $l/\lambda \le 1$, (ii) for $\hbar\Omega > kT$, and (iii) if the plasma particles are degenerated, i.e., $n\lambda^3 > 1$ with n being the density.

Furthermore, a quantum treatment of the collision integral avoids the divergencies occurring in classical theories. Thus no cutting procedure will be necessary.

In this paper, we develop a quantum kinetic theory of strongly correlated plasmas in laser fields which fully includes these phenomena. As the starting point, we use the Kadanoff-Baym equations for the two-time correlation functions of charged particles in an electromagnetic field. These equations are sufficiently general to account for all many-body effects of interest. Moreover, they allow to develop the theory in a highly consistent form where fundamental properties, such as conservation of total energy are satisfied.

II. KADANOFF-BAYM EQUATIONS. GAUGE-INVARIANT GREEN'S FUNCTIONS

Equilibrium and nonequilibrium properties of strongly correlated plasmas are successfully described using the

method of real-time Green's functions. In this framework, the nonequilibrium plasma state is given by the two-time correlation functions which are averages over creation and annihilation operators $\psi^\dagger$ and ψ

$$g_a^>(1,1') = \frac{1}{i\hbar}\langle\psi_a(1)\psi_a^\dagger(1')\rangle,$$

$$g_a^<(1,1') = -\frac{1}{i\hbar}\langle\psi_a^\dagger(1')\psi_a(1)\rangle, \tag{1}$$

where $1 \equiv (\mathbf{r}_1, t_1, s_1^3)$, and a labels the particle species. $g_a^\gtrless$ contain the complete dynamical and statistical information. Their time evolution in an electromagnetic field is determined by the Kadanoff-Baym equations [20,21]

$$\left[i\hbar\frac{\partial}{\partial t_1} - \frac{1}{2m_a}\left(\frac{\hbar}{i}\nabla_1 - \frac{e_a}{c}\mathbf{A}(1)\right)^2 - e_a\phi(1)\right]g_a^\gtrless(1,1')$$

$$+ \int d\bar{\mathbf{r}}_1 \Sigma_a^{HF}(1,\bar{\mathbf{r}}_1 t_1)g_a^\gtrless(\bar{\mathbf{r}}_1 t_1, 1')$$

$$= \int_{t_0}^{t_1} d\bar{1} \left[\Sigma_a^>(1,\bar{1}) - \Sigma_a^<(1,\bar{1})\right] g_a^\gtrless(\bar{1},1')$$

$$- \int_{t_0}^{t_1'} d\bar{1} \Sigma_a^\gtrless(1,\bar{1}) \left[g_a^>(\bar{1},1') - g_a^<(\bar{1},1')\right], \tag{2}$$

which have to be fulfilled together with the adjoint equation. Here t_0 denotes the initial time where the system is assumed to be uncorrelated (otherwise, the equations have to be supplemented with an initial correlation contribution to Σ_a, see Ref. [22]). $\Sigma_a^\gtrless$ are the self-energies which will be discussed below. For the following derivations, it is useful to introduce, in addition, the retarded and advanced Green's functions $g_a^{R/A}(1,1') = \pm\Theta(\pm(t_1 - t_1'))[g_a^>(1,1') - g_a^<(1,1')]$ which obey the simpler equations

$$\left[i\hbar\frac{\partial}{\partial t_1} - \frac{1}{2m_a}\left(\frac{\hbar}{i}\nabla_1 - \frac{e_a}{c}\mathbf{A}(1)\right)^2 - e_a\phi(1)\right]g_a^{R/A}(1,1')$$

$$- \int d2 \Sigma_a^{R/A}(1,2)g_a^{R/A}(2,1') = \delta(1-1'). \tag{3}$$

In these equations, the electromagnetic field is given by the vector and scalar potentials $\mathbf{A}$ and ϕ, and it will be treated classically. In the following, we will use microscopic and macroscopic time and space variables being defined as

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_1', \quad \mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_1')/2,$$

$$\tau = t_1 - t_1', \quad t = (t_1 + t_1')/2.$$

From the Kadanoff-Baym equations (2), there follows immediately the equation of motion for the time-diagonal part of the correlation functions, i.e., for the Wigner distribution function which is given by

$$-i\hbar g_a^<(\mathbf{p},\mathbf{R};t_1,t_1')|_{t_1=t_1'} = f_a(\mathbf{p},\mathbf{R},t).$$

To this end, we consider Eq. (2) for equal times, $t_1 = t_1' = t$, introduce the variables $\mathbf{R}$ and $\mathbf{r}$, and obtain, after Fourier transformation with respect to $\mathbf{r}$, for the spatially homogeneous case,

$$\frac{\partial}{\partial t}f_a(\mathbf{p},t) = 2\,\text{Re}\int_{t_0}^t d\bar{t}\{g_a^>(\mathbf{p};t,\bar{t})\,\Sigma_a^<(\mathbf{p};\bar{t},t)$$

$$- g_a^<(\mathbf{p};t,\bar{t})\,\Sigma_a^>(\mathbf{p};\bar{t},t)\}. \tag{4}$$

This is an exact equation and, therefore, well suited for deriving generalized kinetic equations. To obtain explicit expressions for the collision integral, one has to solve two problems.

(1) It is necessary to find appropriate approximations for the self energy. For this, there exist standard approximations, such as the statically screened Born approximation, the random phase approximation and the T-matrix approximation.

(2) To come to a closed equation for the Wigner function f_a, the correlation functions $g_a^\gtrless$ have to be expressed as functionals of f_a (reconstruction problem). This problem can be solved approximately on the basis of the generalized Kadanoff-Baym ansatz (GKBA) of Lipavský et al. [23], see Sec. III.

To make the derivations transparent, we will consider below the simplest approximation for the self-energies—the statically screened Born approximation, thereby focusing on the modifications introduced by the time-dependent electric field. Further, we will apply the GKBA in its generalization to time-dependent fields [24].

It is well known that the electromagnetic field can be introduced in various ways (gauges) what essentially affects the explicit form of kinetic equations. This becomes a particular problem if one considers approximations to the kinetic equations, such as retardation or gradient expansions, which look different in different gauges. To avoid these difficulties, we will formulate the theory in terms of correlation functions which are made explicitly gauge invariant. The Kadanoff-Baym equations (2) remain covariant under gauge transformations, i.e., under the following transformations of the potentials and field operators:

$$A_\mu'(x) = A_\mu(x) - \partial_\mu\chi(x), \quad \psi_a'(x) = e^{(i/\hbar)(e_a/c)\chi(x)}\psi_a(x), \tag{5}$$

where we use covariant four-vector notation with $A_\mu = (c\phi,\mathbf{A})$, $x_\mu = (c\tau,\mathbf{r})$, $X_\mu = (ct,\mathbf{R})$, $a_\mu b^\mu = a_0 b_0 - \mathbf{a}\cdot\mathbf{b}$ etc. The corresponding gauge transform of the Green's functions leads to

$$g_a'(x,X) = e^{(i/\hbar)(e_a/c)[\chi(X+x/2) - \chi(X-x/2)]}g_a(x,X). \tag{6}$$

Following an idea of Fujita [25], we now introduce a gauge-invariant Green's function $g(\mathbf{k},X)$ which is given by the modified Fourier transform

$$g_a(\mathbf{k},X)=\int\frac{d^4x}{(2\pi)^4}\exp\left\{i\int_{-1/2}^{1/2}d\lambda\,x_\mu\right.$$

$$\left.\times\left[k^\mu+\frac{e_a}{c}A^\mu(X+\lambda x)\right]\right\}g_a(x,X),\qquad(7)$$

where use has been made of the identity

$$\chi\left(X+\frac{x}{2}\right)-\chi\left(X-\frac{x}{2}\right)=\int_{-1/2}^{1/2}d\lambda\,\frac{d}{d\lambda}\chi(X+\lambda x)$$

$$=x_\mu\partial^\mu\int_{-1/2}^{1/2}d\lambda\,\chi(X+\lambda x).$$

Indeed, one readily confirms that under any gauge transform (5), the phase factors cancel, and $g'(\mathbf{k},X)\equiv g(\mathbf{k},X)$, [24].

In the following, we focus on spatially homogeneous electric fields given in vector potential gauge by

$$A_0=\phi=0,\quad \mathbf{A}=-c\int_{-\infty}^t d\bar t\,\mathbf{E}(\bar t).\qquad(8)$$

In this case, relation (7) simplifies to

$$g_a(\mathbf{k},\omega;\mathbf{R},t)=\int d\tau d\mathbf{r}\exp\left[i\,\omega\tau-\frac{i}{\hbar}\mathbf{r}\cdot\left(\mathbf{k}+\frac{e_a}{c}\right.\right.$$

$$\left.\left.\times\int_{t-\tau/2}^{t+\tau/2}\frac{dt'}{\tau}\mathbf{A}(t')\right)\right]g_a(\mathbf{r},\tau;\mathbf{R},t),\quad(9)$$

which means that the gauge-invariant Green's function $g(\mathbf{k})$ follows from the Wigner transformed function $g_a(\mathbf{p})$ by replacing the canonical momentum $\mathbf{p}$ by the gauge-invariant kinematic momentum $\mathbf{k}$ according to

$$\mathbf{p}=\mathbf{k}+\frac{e_a}{c}\int_{t-\tau/2}^{t+\tau/2}dt'\frac{\mathbf{A}(t')}{\tau}.\qquad(10)$$

In particular, for a harmonic electric field,

$$\mathbf{E}(t)=\mathbf{E}_0\cos\Omega t,\quad \mathbf{A}(t)=-\frac{c\mathbf{E}_0}{\Omega}\sin\Omega t,\qquad(11)$$

the substitution for the momentum takes the form

$$\mathbf{p}=\mathbf{k}+\frac{2}{\tau}\frac{\mathbf{E}_0}{\Omega^2}\sin\Omega t\sin\frac{\Omega\tau}{2}.\qquad(12)$$

For the derivations below, we will need the gauge invariant Fourier transform of the convolution of two functions

$$I(\mathbf{r}_1-\mathbf{r}_1';t_1,t_1')=\int d\bar t\,d\bar{\mathbf{r}}\,B(\mathbf{r}_1-\bar{\mathbf{r}};t_1,\bar t)\,C(\bar{\mathbf{r}}-\mathbf{r}_1';\bar t,t_1').$$
$$(13)$$

After straightforward manipulations which involve the back transform of Eq. (9), we arrive at

$$I(\mathbf{k};t_1,t_1')=\int d\bar t\,B\left(\mathbf{k}+\frac{e_a}{c}\int_{t_1'}^{t_1}dt''\frac{\mathbf{A}(t'')}{t_1-t_1'}\right.$$

$$-\frac{e_a}{c}\int_{\bar t}^{t_1}dt''\frac{\mathbf{A}(t'')}{t_1-\bar t};t_1,\bar t\right)$$

$$\times C\left(\mathbf{k}+\frac{e_a}{c}\int_{t_1'}^{t_1}dt''\frac{\mathbf{A}(t'')}{t_1-t_1'}\right.$$

$$\left.-\frac{e_a}{c}\int_{t_1'}^{\bar t}dt''\frac{\mathbf{A}(t'')}{\bar t-t_1'};\bar t,t_1'\right).\qquad(14)$$

In the equal-time limit, $t_1=t_1'=t$, the momentum arguments of B and C are identical, equal to

$$\mathbf{k}+\frac{e_a}{c}\mathbf{A}(t)-\frac{e_a}{c}\int_{\bar t}^t dt''\frac{\mathbf{A}(t'')}{t-\bar t}.$$

III. GAUGE-INVARIANT PROPAGATOR. GENERALIZED KADANOFF-BAYM ANSATZ

To solve the reconstruction problem, we need expressions for the retarded and advanced Green's functions. We first determine these quantities for free particles in an electromagnetic field. In this case, Eq. (3) simplifies to

$$\left[i\hbar\frac{\partial}{\partial t_1}-\frac{1}{2m_a}\left\{\mathbf{p}-\frac{e_a}{c}\mathbf{A}(t_1)\right\}^2\right]g_a^{R/A}(\mathbf{p};t_1,t_1')=\delta(t_1-t_1'),$$

with the solution

$$g_a^{R/A}(\mathbf{p};\tau,t)=\mp\frac{i}{\hbar}\Theta(\pm\tau)\exp\left\{-\frac{i}{\hbar}\int_{t-\tau/2}^{t+\tau/2}dt'\right.$$

$$\left.\times\left[\mathbf{p}-\frac{e_a}{c}\mathbf{A}(t')\right]^2/2m_a\right\},\qquad(15)$$

which yields for the spectral function, $a_a\equiv i\hbar(g_a^R-g_a^A)$,

$$a_a(\mathbf{p};\tau,t)=\exp\left\{-\frac{i}{\hbar}\int_{t-\tau/2}^{t+\tau/2}dt'\left[\mathbf{p}-\frac{e_a}{c}\mathbf{A}(t')\right]^2/2m_a\right\}.$$
$$(16)$$

The functions (15) and (16) depend on the canonic momentum $\mathbf{p}$ and are gauge dependent. As pointed out before, it is useful to introduce instead gauge-invariant functions by applying the transform (7). A simple calculation yields

$$g_a^{R/A}(\mathbf{k};\tau,t)=\mp\frac{i}{\hbar}\Theta(\pm\tau)\exp\left\{-\frac{i}{\hbar}\left[\frac{k^2}{2m_a}\tau+S_a(\mathbf{A};\tau,t)\right]\right\},$$
$$(17)$$

with

$$S_a(\mathbf{A};\tau,t)=\frac{e_a^2}{2m_ac^2}\left[\int_{t-\tau/2}^{t+\tau/2}dt'A^2(t')\right.$$

$$\left.+\frac{1}{\tau}\left(\int_{t-\tau/2}^{t+\tau/2}dt'\mathbf{A}(t')\right)^2\right].\qquad(18)$$

For a harmonic time dependence of the field as given by Eq. (11), the time integrations in S can be performed, and simple trigonometric relations lead to [26]

$$S_a(\mathbf{A};\tau,t)=\varepsilon_a^{\text{pond}}\,\tau\left[1-\frac{\sin\Omega\tau\cos2\Omega t}{\Omega\tau}\right.$$
$$\left.+\frac{8\sin^2\Omega t\sin^2(\Omega\tau/2)}{(\Omega\tau)^2}\right], \quad (19)$$

where we introduced the familiar ponderomotive potential

$$\varepsilon_a^{\text{pond}}=\frac{e_a^2E_0^2}{4m_a\Omega^2}, \quad (20)$$

which is just the time-averaged kinetic energy of a free particle in the harmonic field.

Now we turn to the solution of the reconstruction problem. Due to the expected retardation effects, the common Kadanoff-Baym ansatz is not applicable here. As mentioned above, a more general solution which properly takes into account retardation (memory) effects has been proposed by Lipavský et al. [23]:

$$g_a^<(\mathbf{p};t_1,t_1')=i\hbar g_a^R(\mathbf{p};t_1,t_1')g_a^<(\mathbf{p};t_1',t_1')$$
$$-i\hbar g_a^<(\mathbf{p};t_1,t_1)g_a^A(\mathbf{p};t_1,t_1'). \quad (21)$$

This represents an exact relation in quasiparticle approximation assuming static self-energies. However, the ansatz (21) is often used approximately with more general propagators $g_a^{R/A}$ [24].

To transform this relation into a gauge-invariant form, we start from its coordinate representation which, for the case $t_1\geq t_1'$, reads

$$g_a^<(\mathbf{r}_1-\mathbf{r}_1';t_1,t_1')=i\hbar\int d\bar{\mathbf{r}}\,g_a^R(\mathbf{r}_1-\bar{\mathbf{r}};t_1,t_1')$$
$$\times g_a^<(\bar{\mathbf{r}}-\mathbf{r}_1';t_1',t_1'). \quad (22)$$

Gauge-invariant Fourier transform of Eq. (22) and use of the back transforms for g_a^R and $g_a^<$ leads directly to

$$g_a^<(\mathbf{k};t_1,t_1')=-g_a^R(\mathbf{k};t_1,t_1')f_a\left[\mathbf{k}-\frac{e_a}{c}\mathbf{A}(t_1')\right.$$
$$\left.+\frac{e_a}{c}\int_{t_1'}^{t_1}d\bar{t}\frac{\mathbf{A}(\bar{t})}{t_1-t_1'};t_1'\right], \quad (23)$$

which is the gauge-invariant generalization of the GKBA. For $t_1<t_1'$, the second part of Eq. (21) applies which is transformed analogously. The same transformations are performed for $g_a^>$, leading to the result (23) with f_a being replaced by $1-f_a$.

IV. KINETIC EQUATION FOR QUANTUM PARTICLES IN AN ELECTROMAGNETIC FIELD

We now come back to the time-diagonal limit of the Kadanoff-Baym equations, cf. Eqs. (2) and (4), and derive

the quantum kinetic equation for a plasma in a laser field. Again, it is advantageous to derive this equation for the gauge-invariant Wigner distribution. To this end, we take the Fourier transform (9) of the time-diagonal Kadanoff-Baym equation and make use of relation (14) for the special case $t_1=t_1'$ and obtain

$$\frac{\partial}{\partial t}f_a(\mathbf{k}_a,t)+e_a\mathbf{E}(t)\cdot\boldsymbol{\nabla}_\mathbf{k}f_a(\mathbf{k}_a,t)$$
$$=2\,\text{Re}\int_{t_0}^t d\bar{t}\,(\{g_a^>,\Sigma_a^<\}-\{g_a^<,\Sigma_a^>\}), \quad (24)$$

where we denoted

$$\{g_a,\Sigma_a\}=g_a\left(\mathbf{k}+\frac{e_a}{c}\mathbf{A}(t)-\frac{e_a}{c}\int_{\bar{t}}^t dt'\frac{\mathbf{A}(t')}{t-\bar{t}};t,\bar{t}\right)$$
$$\times\Sigma_a\left(\mathbf{k}+\frac{e_a}{c}\mathbf{A}(t)-\frac{e_a}{c}\int_{\bar{t}}^t dt'\frac{\mathbf{A}(t')}{t-\bar{t}};\bar{t},t\right).$$

For the collision term, we use here self-energies in the simple second Born approximation. Starting from the familiar expression in coordinate representation, a straightforward calculation leads to the gauge-invariant result

$$\Sigma_a^{\gtrless}(\mathbf{k}_a;t_1,t_1')=\sum_b\int\frac{d\mathbf{k}_b d\bar{\mathbf{k}}_a d\bar{\mathbf{k}}_b}{(2\pi\hbar)^9}|V_{ab}(\mathbf{k}_a-\bar{\mathbf{k}}_a)|^2$$
$$\times(2\pi\hbar)^3\delta(\mathbf{k}_a+\mathbf{k}_b-\bar{\mathbf{k}}_a-\bar{\mathbf{k}}_b)$$
$$\times\hbar^2 g_a^{\gtrless}(\bar{\mathbf{k}}_a;t_1,t_1')g_b^{\gtrless}(\bar{\mathbf{k}}_b;t_1,t_1')$$
$$\times g_b^{\lessgtr}(\mathbf{k}_b;t_1',t_1). \quad (25)$$

The correlation functions $g_a^{\gtrless}$ are expressed by the Wigner functions with the help of the GKBA (23). Using for the retarded and advanced Green's functions the uncorrelated expressions (17) and taking into account the property $S_a(\mathbf{A},-\tau,t)=-S_a(\mathbf{A},\tau,t)$, a lengthy but straightforward calculation leads to the following kinetic equation ($t_1=t_1'=t$)

$$\left\{\frac{\partial}{\partial t}+e_a\mathbf{E}(t)\cdot\boldsymbol{\nabla}_{k_a}\right\}f_a(\mathbf{k}_a,t)=\sum_b I_{ab}(\mathbf{k}_a,t) \quad (26)$$

with the collision integral

$$I_{ab}(\mathbf{k}_a,t)=2\int\frac{d\mathbf{k}_b d\bar{\mathbf{k}}_a d\bar{\mathbf{k}}_b}{(2\pi\hbar)^6}\frac{1}{\hbar^2}|V_{ab}(\mathbf{k}_a-\bar{\mathbf{k}}_a)|^2$$
$$\times\delta(\mathbf{k}_a+\mathbf{k}_b-\bar{\mathbf{k}}_a-\bar{\mathbf{k}}_b)$$
$$\times\int_{t_0}^t d\bar{t}\,\text{Re}\exp\left\{\frac{i}{\hbar}[(\epsilon_{ab}-\bar{\epsilon}_{ab})(t-\bar{t})\right.$$
$$\left.-(\mathbf{k}_a-\bar{\mathbf{k}}_a)\cdot\mathbf{R}_{ab}(t,\bar{t})]\right\}\{\bar{f}_a\bar{f}_b[1-f_a][1-f_b]$$
$$-f_af_b[1-\bar{f}_a][1-\bar{f}_b]\}_{|\bar{t}}, \quad (27)$$

where we denoted $\epsilon_{ab} = \epsilon_a + \epsilon_b$, $\epsilon_a = p_a^2/2m_a$ and $f_a = f_a[\mathbf{k}_a + \mathbf{Q}_a(t,\bar{t}),\bar{t}]$, and the quantities $\mathbf{Q}_a$ and $\mathbf{R}_{ab}$ are defined by Eqs. (28) and (29) below.

This is a rather general kinetic equation which describes two-particle collisions in a weakly coupled quantum plasma in the presence of a spatially homogeneous time-dependent field. It generalizes previous results obtained for classical plasmas [4–6].

The time-dependent field modifies the collision integral in several ways.

(1) The momentum arguments of the distribution functions are

$$\mathbf{k}_a + \mathbf{Q}_a(t,\bar{t}), \quad \text{with} \quad \mathbf{Q}_a(t,\bar{t}) \equiv -e_a \int_{\bar{t}}^{t} dt' \, \mathbf{E}(t'), \quad (28)$$

i.e., they contain an additional retardation given by $\mathbf{Q}_a$, the intracollisional field contribution to the momentum. It describes the gain of momentum in the time interval $t - \bar{t}$ due to the field. In the case of a harmonic field given by Eq. (11), we have

$$\mathbf{Q}_a(t,\bar{t}) = -\frac{e_a \mathbf{E}_0}{\Omega}(\sin \Omega t - \sin \Omega \bar{t}).$$

The result for a static field [10] is readily recovered by taking the limit $\Omega \to 0$, i.e., $\mathbf{Q}_a^{st} = -e_a \mathbf{E}_0 (t - \bar{t})$.

(2) Another modification occurs in the exponent under the time integral which essentially governs the energy balance in a two-particle collision: In addition to the usual collisional energy broadening (which has the form $\cos\{[\epsilon_{ab} - \bar{\epsilon}_{ab}](t - \bar{t})/\hbar\}$), there appears a field-dependent broadening. This effect is determined by the change of the distance between particles a and b due to the field given by

$$\mathbf{R}_{ab}(t,\bar{t}) = \left(\frac{e_a}{m_a} - \frac{e_b}{m_b}\right) \int_{\bar{t}}^{t} dt' \int_{t'}^{t} dt'' \, \mathbf{E}(t''). \quad (29)$$

We get for harmonic fields

$$\mathbf{R}_{ab}(t,\bar{t}) = \left(\frac{e_a}{m_a} - \frac{e_b}{m_b}\right)\left[\frac{\mathbf{E}_0 \cdot (t - \bar{t})}{\Omega}\sin\Omega t\right.$$
$$\left. + \frac{\mathbf{E}_0}{\Omega^2}(\cos\Omega t - \cos\Omega\bar{t})\right]. \quad (30)$$

It is clear that the field has no effect on scattering of identical particles, $\mathbf{R}_{aa} \equiv 0$. [Of course, the scattering rates (collision frequency) of identical particles will be modified by the field indirectly via the distribution functions.] For static fields, it follows $\mathbf{R}_{ab}^{st}(t,\bar{t}) = (e_a/m_a - e_b/m_b)\mathbf{E}_0/2(t-\bar{t})^2$.

(3) A third important effect is the nonlinear (exponential) dependence of the collision term in Eq. (27) on the field strength which we discuss more in detail below.

As mentioned above, the kinetic equation (26) represents a generalized version of kinetic equations used in dense plasma physics. Neglection of quantum effects leads to the well-known classical kinetic equation derived by Silin [4]. On the other hand, for static fields Eq. (26) reduces to the kinetic equation given in Ref. [10]. Finally, in the zero-field

case, we get the non-Markovian form of the well-known quantum Landau equation [6].

Now, let us consider the consequences of the nonlinear field dependence in the kinetic equation (38). This nonlinearity gives rise to interesting physical processes including generation of higher field harmonics and emission/absorption of multiple photons in two-particle scattering in the case of high field strength. To show this, we expand the spectral kernel of the collision integrals I_{ab} [second line in Eq. (27)] into a Fourier series making use of the familiar relation

$$e^{\pm iz\cos\Omega t} = \sum_{n=-\infty}^{\infty} (\pm i)^n J_n(z) e^{\mp in\Omega t}, \quad (31)$$

where J_n denotes the Bessel function of nth order. As a result, the collision integral in the kinetic equation (26) transforms according to

$$I_{ab}(\mathbf{k}_a,t) = 2\,\mathrm{Re} \int \frac{d\mathbf{k}_b d\bar{\mathbf{k}}_a d\bar{\mathbf{k}}_b}{(2\pi\hbar)^6} \frac{1}{\hbar^2} |V_{ab}(\mathbf{k}_a - \bar{\mathbf{k}}_a)|^2$$

$$\times \delta(\mathbf{k}_a + \mathbf{k}_b - \bar{\mathbf{k}}_a - \bar{\mathbf{k}}_b)$$

$$\times \sum_{n=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} (i)^l J_n\left(\frac{\mathbf{q} \cdot \mathbf{w}_{ab}^0}{\hbar\Omega}\right) J_{n+l}\left(\frac{\mathbf{q} \cdot \mathbf{w}_{ab}^0}{\hbar\Omega}\right)$$

$$\times [\cos(l\Omega t) - i\sin(l\Omega t)] \int_{t_0}^{t} d\bar{t}\,\exp\left\{\frac{i}{\hbar}[\epsilon_{ab} - \bar{\epsilon}_{ab}\right.$$

$$\left. - \mathbf{q} \cdot \mathbf{w}_{ab}(t) + n\hbar\Omega]\right\}(t-\bar{t})\{\bar{f}_a \bar{f}_b [1 - f_a][1$$

$$- f_b] - f_a f_b [1 - \bar{f}_a][1 - \bar{f}_b]\}_{|\bar{t}}, \quad (32)$$

where we denoted

$$\mathbf{q} \equiv \mathbf{k}_a - \bar{\mathbf{k}}_a = \bar{\mathbf{k}}_b - \mathbf{k}_b, \quad \mathbf{w}_{ab}(t) \equiv \mathbf{v}_a(t) - \mathbf{v}_b(t),$$

$$\mathbf{w}_{ab}^0 \equiv \mathbf{v}_a^0 - \mathbf{v}_b^0,$$

$$\mathbf{v}_a(t) = \mathbf{v}_a^0 \sin\Omega t, \quad \mathbf{v}_a^0 \equiv \frac{e_a \mathbf{E}_0}{m_a\Omega}. \quad (33)$$

Obviously, $\mathbf{v}_a$ is the velocity of a classical particle of charge e_a in the periodic field. Separating real and imaginary parts, we get

$$I_{ab}(\mathbf{k}_a,t) = \sum_n \sum_{l=\text{even}} i^l[\mathrm{Re}\,I_{ab}^{nl}\cos l\Omega t + \mathrm{Im}\,I_{ab}^{nl}\sin l\Omega t]$$

$$+ \sum_n \sum_{l=\text{odd}} i^{l+1}[\mathrm{Im}\,I_{ab}^{nl}\cos l\Omega t$$

$$- \mathrm{Re}\,I_{ab}^{nl}\sin l\Omega t], \quad (34)$$

where the summations over n and l run from $-\infty$ to $+\infty$ and the expansion coefficients I_{ab}^{nl} are given by

$$I_{ab(ka,t)}^{nl} = 2 \int \frac{d\mathbf{k}_b d\bar{\mathbf{k}}_a d\bar{\mathbf{k}}_b}{(2\pi\hbar)^6} \frac{1}{\hbar^2} |V_{ab}(\mathbf{k}_a - \bar{\mathbf{k}}_a)|^2$$

$$\times \delta(\mathbf{k}_a + \mathbf{k}_b - \bar{\mathbf{k}}_a - \bar{\mathbf{k}}_b)$$

$$\times \int_{t_0}^t d\bar{t} J_n\left(\frac{\mathbf{q}\cdot\mathbf{w}_{ab}^0}{\hbar\Omega}\right) J_{n-l}\left(\frac{\mathbf{q}\cdot\mathbf{w}_{ab}^0}{\hbar\Omega}\right) \exp\left\{\frac{i}{\hbar}[\epsilon_{ab} - \bar{\epsilon}_{ab}\right.$$

$$\left. - \mathbf{q}\cdot\mathbf{w}_{ab}(t) + n\hbar\Omega](t-\bar{t})\right\} \{\bar{f}_a\bar{f}_b[1-f_a][1-f_b]$$

$$- f_a f_b[1-\bar{f}_a][1-\bar{f}_b]\}_{|\bar{t}}. \quad (35)$$

This representation of the collision integral allows for a transparent physical interpretation of the binary scattering process.

(1) In a strong periodic field, Coulomb collisions with a momentum change $\mathbf{q}$ give rise to the generation of higher harmonics of the field (sums over l). This has already been shown by Silin for classical plasmas. These terms are important on very short time scales, whereas they do not contribute to transport quantities which are averaged over times larger than the field period $2\pi/\Omega$.

(2) Furthermore, it is obvious that collisions in strong harmonic fields are accompanied by emission and absorption of multiple photons, cf. the sum over n. Indeed, if the retardation in the distributions is omitted, and the initial time is shifted to minus infinity, the time integration in the collision term can be performed, giving rise to an energy delta function

$$\delta[\epsilon_{ab} - \bar{\epsilon}_{ab} + \mathbf{q}\cdot\mathbf{w}_{ab}(t) - n\hbar\Omega].$$

This function describes the energy conservation in a two-particle scattering process in the presence of a periodic electric field, which leads to multiphoton emission and absorption.

For high frequency Ω, the collision integral I_{ab} given by the expression (27) can be simplified by averaging over a period of the oscillating field. The spectral kernel (34) then reduces to

$$\text{Re} \exp\left\{\frac{i}{\hbar}[(\epsilon_{ab} - \bar{\epsilon}_{ab})(t-\bar{t}) - \mathbf{q}\cdot\mathbf{R}_{ab}(t,\bar{t})]\right\}$$

$$= \sum_{n=-\infty}^{+\infty} \cos\left\{\frac{1}{\hbar}[\epsilon_{ab} - \bar{\epsilon}_{ab} - \mathbf{q}\cdot\mathbf{w}_{ab}(t) + n\hbar\Omega]\right.$$

$$\left. \times (t-\bar{t})\right\} J_n^2\left(\frac{\mathbf{q}\cdot\mathbf{w}_{ab}^0}{\hbar\Omega}\right). \quad (36)$$

Using this expression in Eq. (27), we find

$$\left\{\frac{\partial}{\partial t} + e_a\mathbf{E}(t)\cdot\nabla_{k_a}\right\} f_a(\mathbf{k}_a,t) = \sum_b I_{ab}(\mathbf{k}_a,t) \quad (37)$$

with the collision integral

$$I_{ab}(\mathbf{k}_a,t) = 2 \int \frac{d\mathbf{k}_b d\bar{\mathbf{k}}_a d\bar{\mathbf{k}}_b}{(2\pi\hbar)^6} \frac{1}{\hbar^2} |V_{ab}(\mathbf{k}_a - \bar{\mathbf{k}}_a)|^2$$

$$\times \delta(\mathbf{k}_a + \mathbf{k}_b - \bar{\mathbf{k}}_a - \bar{\mathbf{k}}_b)$$

$$\times \sum_n J_n^2\left(\frac{\mathbf{q}\cdot\mathbf{w}_{ab}^0}{\hbar\Omega}\right) \int_{t_0}^t d\bar{t}\cos\left[\frac{1}{\hbar}(\epsilon_{ab} - \bar{\epsilon}_{ab}\right.$$

$$\left. - \mathbf{q}\cdot\mathbf{w}_{ab}(t) + n\hbar\Omega)(t-\bar{t})\right] \{\bar{f}_a\bar{f}_b[1-f_a]$$

$$\times [1-f_b] - f_a f_b[1-\bar{f}_a][1-\bar{f}_b]\}_{|\bar{t}}. \quad (38)$$

This quantum kinetic equation is a special case of Eq. (26). It can be applied to the important case of dense plasmas in strong high-frequency laser fields. For classical plasmas, such an equation was given by Klimontovich [6].

V. APPLICATION TO STRONG LASER PULSES

Let us consider the case of a laser pulse with the electric field given by $\mathbf{E}^L(t) = \mathbf{E}_0^L(t)[e^{i\Omega t} + e^{-i\Omega t}]$, where $\mathbf{E}_0^L(t)$ is the (real) pulse envelope. In a strong field, we expect that also the electron distribution will be periodically modulated with the frequency Ω and, possibly, with higher harmonics. We, therefore, look for solutions of the kinetic equation (26) of the form

$$f_a(\mathbf{k}_a,t) = \sum_{l=-\infty}^{\infty} f_{a,l}(\mathbf{k}_a,t) e^{-il\Omega t}, \quad (39)$$

where $f_{a,l}$ are complex amplitudes. Inserting Eq. (39) into Eq. (38) and neglecting, for a moment, polarization induced modifications of the electric field, we obtain a system of coupled equations for the distribution harmonics

$$\left\{-il\Omega + \frac{\partial}{\partial t}\right\} f_{a,l} + e_a\mathbf{E}_0^L\cdot\nabla_{k_a}(f_{a,l-1} + f_{a,l+1}) = \sum_b I_{ab,l},$$

$$(40)$$

supplemented with the initial conditions given at time t_0 sufficiently long before the pulse, $\mathbf{E}_0^L(\mathbf{k},t_0) = 0$ and $f_{a,l}(\mathbf{k}_a,t_0) = f_a^0(\mathbf{k}_a)\delta_{l,0}$. The Fourier components of the collision integrals $I_{ab,l}$ are defined in analogy to Eq. (39). The harmonics spectrum $I_{ab,l}$ is rather complicated since it depends on the spectrum of the retardation kernel $\mathbf{R}_{ab}$, but also on the spectrum of the distributions. We consider first the effect of $\mathbf{R}_{ab}$, replacing in the collision integrals f_a by f_a^0 ($l=0$). In this zeroth order approximation, due to the property $\mathbf{R}_{aa} = 0$ (see above), the equal-particle scattering integrals are not oscillating, i.e., $I_{aa} \approx I_{aa,0}$.

On the other hand, for the integrals I_{ab} with $a \neq b$, the expansion coefficients are readily calculated. Using again relation (31), we obtain

$$I_{ab}^0(\mathbf{k}_a,t) = \sum_{l=-\infty}^{\infty} I_{ab,l}^0(\mathbf{k},t) e^{-il\Omega t},$$

$$
I_{ab,l}^0(\mathbf{k}_a,t) = 2(-i)^l \int \frac{d\mathbf{k}_b d\bar{\mathbf{k}}_a d\bar{\mathbf{k}}_b}{(2\pi\hbar)^9} \frac{1}{\hbar^2} |V_{ab}(\mathbf{k}_a - \bar{\mathbf{k}}_a)|^2 (2\pi\hbar)^3 \delta(\mathbf{k}_a + \mathbf{k}_b - \bar{\mathbf{k}}_a - \bar{\mathbf{k}}_b) \sum_{n=-\infty}^{\infty} J_n\left(\frac{\mathbf{q}\cdot\mathbf{w}_{ab}^0}{\hbar\Omega}\right)
$$

$$
\times \int_{t_0}^t d\bar{t} \left\{ J_{n+l}\left(\frac{\mathbf{q}\cdot\mathbf{w}_{ab}^0}{\hbar\Omega}\right) \exp\left\{ -\frac{i}{\hbar}[\epsilon_{ab} - \bar{\epsilon}_{ab} + \mathbf{q}\cdot\mathbf{w}_{ab}(t) + n\hbar\Omega](t-\bar{t}) \right\} + J_{n-l}\left(\frac{\mathbf{q}\cdot\mathbf{w}_{ab}^0}{\hbar\Omega}\right) \right.
$$

$$
\left. \times \exp\left\{ \frac{i}{\hbar}[\epsilon_{ab} - \bar{\epsilon}_{ab} + \mathbf{q}\cdot\mathbf{w}_{ab}(t) + n\hbar\Omega](t-\bar{t}) \right\} \right\} \{ \bar{f}_{a,0}\bar{f}_{b,0}[1-f_{a,0}][1-f_{b,0}] - f_{a,0}f_{b,0}[1-\bar{f}_{a,0}][1-\bar{f}_{b,0}] \}_{|\bar{t}},
$$

$$
(41)
$$

where the superscript ''0'' underlines that the non-oscillatory parts of the distributions are used. The first order terms $I_{ab,l}^1$ contain the first harmonic of one distribution, e.g., $\bar{f}_{a1}$, while for the other distributions the zeroth order is taken and so on.

Finally, in a strong field, polarization effects become essential. In that case, everywhere the laser field has to be replaced by the total field $\mathbf{E}^L(t) \to \mathbf{E}(t)$ which is the solution of Maxwell's equations. In the homogeneous case, this solution is given by

$$
\mathbf{E}(t) = \mathbf{E}^L(t) - 4\pi \int_{t_0}^t d\bar{t}\ \mathbf{j}(\bar{t}), \tag{42}
$$

where the current is self-consistently given by the distribution functions

$$
\mathbf{j}(t) = \sum_a e_a \int \frac{d\mathbf{k}_a}{(2\pi\hbar)^3} \frac{\mathbf{k}_a}{m_a} \sum_l f_{al}(\mathbf{k}_a,t). \tag{43}
$$

Obviously, the harmonic modulation of the distributions leads directly to the generation of higher field harmonics. The above system of equations fully describes the time-dependent plasma response to the laser pulse. It will be efficient, of course, only for moderate intensities, when the higher field harmonics do not significantly contribute to the collision integral. Otherwise, the field in I_{ab} has to be replaced by a harmonics expansion itself leading to a more complex spectrum $I_{ab,l}$. Then, a direct integration of the collision term (38) seems preferable.

VI. DISCUSSION

Let us briefly discuss the range of applicability of our results. There is no limitation on the frequency Ω, therefore, our kinetic equation is valid for undercritical ($\Omega > \omega_{\mathrm{pl}}$) and overcritical ($\Omega < \omega_{\mathrm{pl}}$) plasmas, for optical and x-ray fields as well. Due to the presented nonrelativistic derivation, the field amplitude must not exceed a maximum value. One readily checks that for the intracollisional field effect to remain nonrelativistic, it is sufficient that the fields are below approximately the interatomic hydrogen field, $E_0^{\max} < 5 \times 10^9$ V/cm, for optical fields. With increasing frequency, $E_0^{\max}$ grows linearly with Ω. As in the case of static fields, increasing the plasma density reduces the effect of the field.

In summary, we have presented in this paper a gauge-invariant derivation of the quantum kinetic equation for dense plasmas in a laser field. Our main result, Eq. (38), generalizes previous work to quantum systems. This equation can be used to calculate the transport properties of a dense plasma in a laser field on arbitrary time scales, i.e., over the whole frequency range. The use of the simple static Born approximation allowed for a very transparent discussion of the influence of the electromagnetic field on the two-particle scattering process. On the other hand, the presented gauge-invariant scheme can be extended straightforwardly to describe relativistic systems or to include more complex many-body effects, such as dynamical screening and femtosecond screening buildup in the presence of a laser pulse. Furthermore, it will be of interest to consider the modification of strong collisions (T-matrix approximation) due to the field, as well as the incorporation of bound states and impact and field ionization. This will allow us to microscopically describe the plasma generation process in the field of a strong laser pulse.

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